Solution of the one-dimensional Newton's equation for a force that depends only on the velocity: $F=F(v)$

Examples of *F(v)*

- Only important examples of *F(v)* in one dimension are *frictional forces*.

- Dry surfaces: $F(v) = \mu N =$ constant

  μ = coefficient friction; N = normal force

- Lubricated surfaces: *F(v)* dependence on *v* may be very complicated
  - Solid-liquid interfaces
  - Solid-gas interfaces

F(v) for lubricated surfaces

- In some cases a good *model* for *F(v)* is given by:

$$F(v) = \mp b v^n$$ n integer (b =constant)

  - *F(v)* proportional to a fixed power of v
  - *F(v)* always opposite to v

    n odd: – sign chosen: $F(v) \propto -v^n$

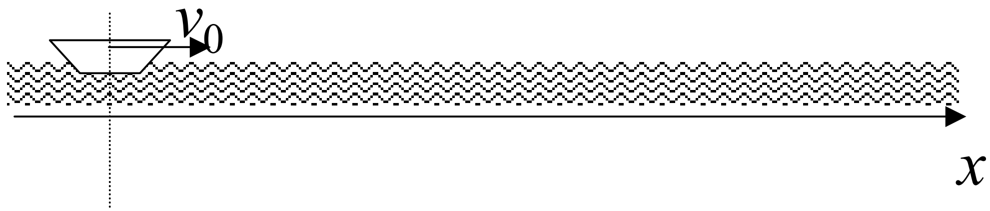
    n even: – *or* + sign chosen: Ensure *F(v)* opposite to v
  - Work done by *F(v)*:

$$W_{x_0 \to x} = \int_{x_0}^{x} F(x)\,dx = \int_{t_0}^{t} \overbrace{F(v)}^{-}\,\overset{+}{v}\,dt < 0, \quad \forall \text{ motion}$$

$W < 0 \Rightarrow$ Force absorbs energy from motion (Non-conservative force)

Example 1: Boat on lake (horizontal motion)

- A motorboat of mass m moving on the surface of a still lake. The engines are suddenly turned off. What is the motion after the engines are turned off?



Engines turned off

$t_0 = 0$

x_0

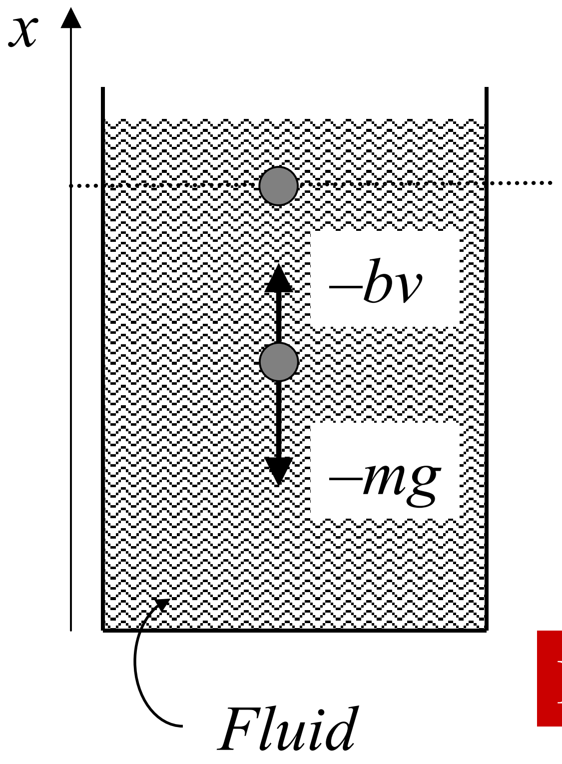
v_0

After engines are turned off the only force the boat is subjected to is the viscous force caused by the friction with water.

Model for viscous force: $n = 1 \Rightarrow F(v) = -bv$

Example 2: Fall through fluid (vertical motion)

- Describe the motion of a spherical object of mass *m* falling through a fluid.



Released $t_0 = 0, x_0, v_0$

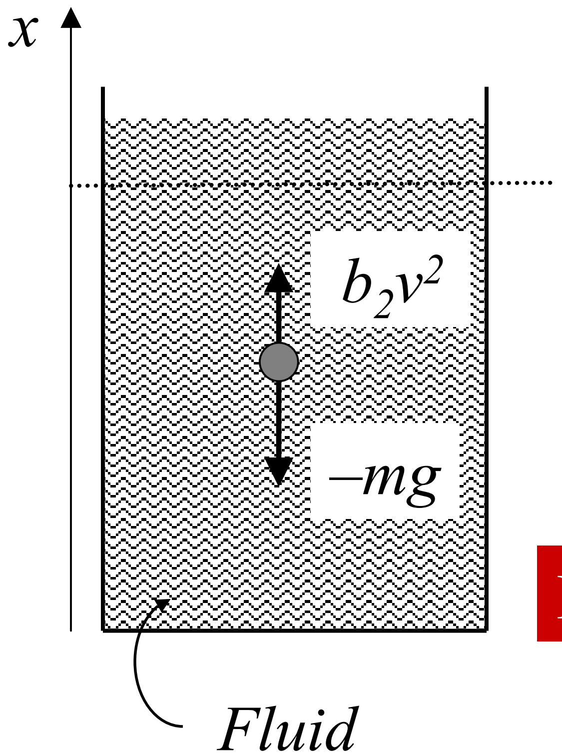
As sphere falls it is subjected to *gravity* and the *viscous force* (opposed to the motion) caused by the friction with the fluid.

Model for viscous force: $n = 1 \Rightarrow F(v) = -bv$

Example 3: Fall through fluid (vertical motion)

(Viscous force quadratic on velocity)

- Describe the motion of a spherical object of mass m falling through a fluid.



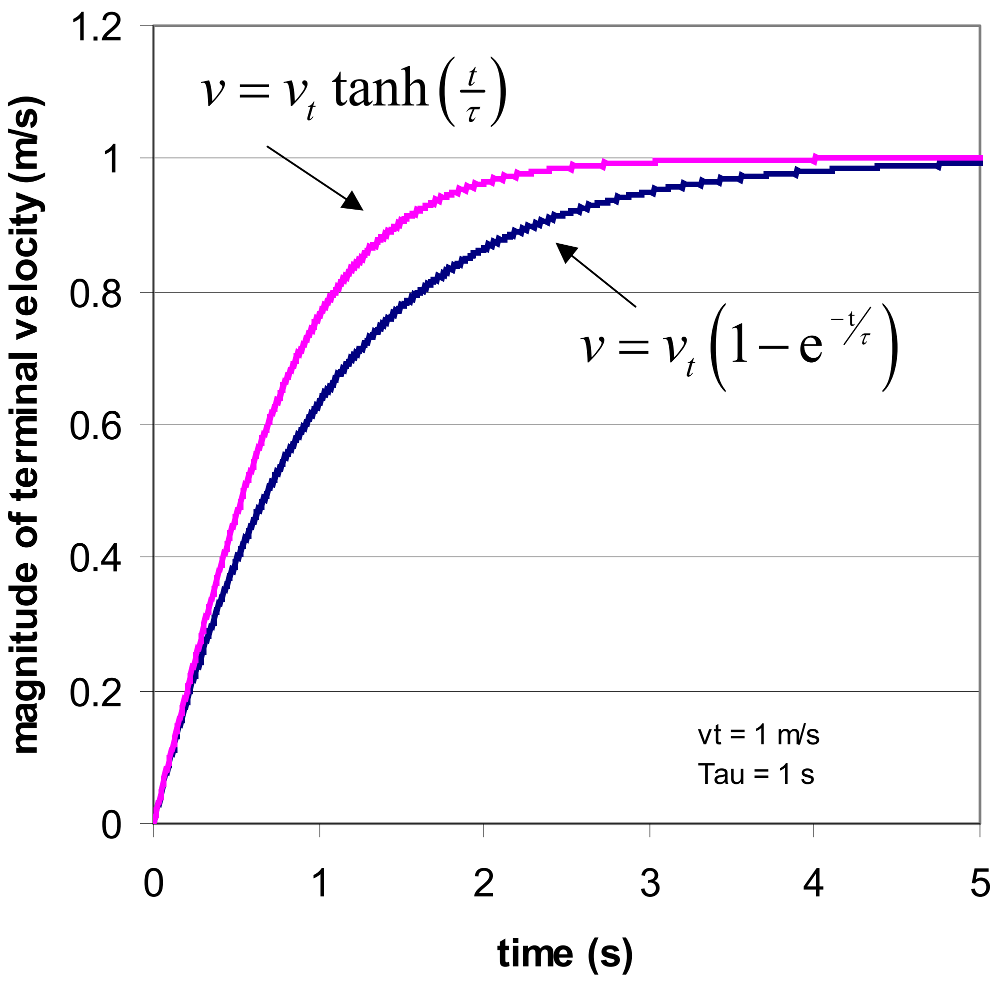
Released $t_0 = 0, x_0, v_0$

$$F(v) = \mp b v^n$$

Model for viscous force: $n = 2 \Rightarrow F(v) = b_2 v^2$

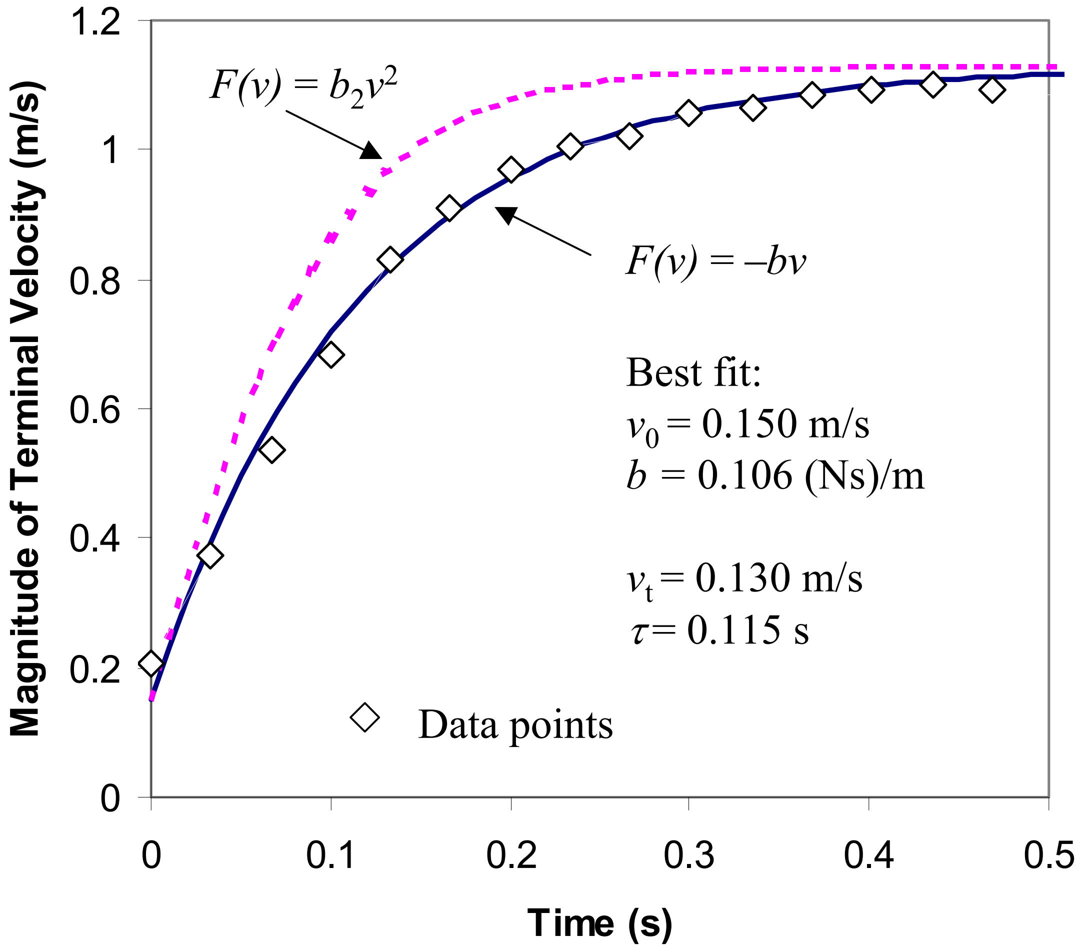
Terminal Velocity *(Fall through fluid)*

$v_0 = 0$



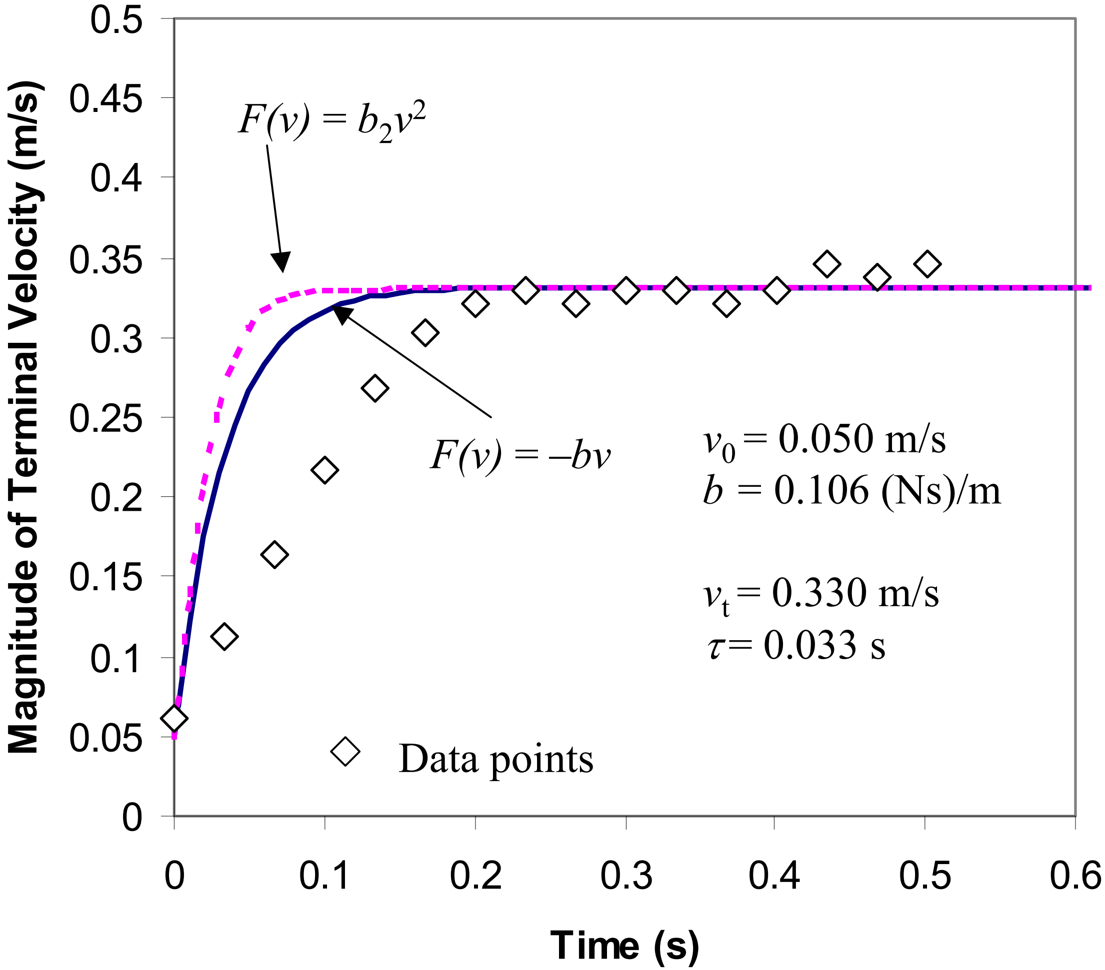
Lead sphere falling through oil

$m = 12.22$ g



Marble falling through oil

$m = 3.72$ g



Reason for discrepancy:

Terminal velocity reached faster than time resolution of data acquisition.